Journal of Universal Computer Science, vol. 16, no. 1 (2010), 3-21
submitted: 1/2/09, accepted: 15/10/09, appeared: 1/1/2010 © J.UCS

Some Views on Information Fusion and Logic Based Approaches in Decision Making under Uncertainty

Yang Xu
(School of Mathematics, Southwest Jiaotong University, Chengdu, Sichuan 610031, China
xuyang@home.swjtu.edu.cn)

Jun Liu
(School of Computing and Mathematics, University of Ulster, Northern Ireland, UK
j.liu@ulster.ac.uk)

Luis Martínez
(School of Computing, Dept. of Computer Science, University of Jaén, 23071 Jáen, Spain
martin@ujaen.es)

Da Ruan
(Belgian Nuclear Research Centre (SCK•CEN), Mol, and Ghent University, Gent, Belgium
druan@sckcen.be; da.ruan@ugent.be)

Abstract: Decision making under uncertainty is a key issue in information fusion and logic based reasoning approaches. The aim of this paper is to show noteworthy theoretical and applicational issues in the area of decision making under uncertainty that have been already done and raise new open research related to these topics pointing out promising and challenging research gaps that should be addressed in the coming future in order to improve the resolution of decision making problems under uncertainty.

Keyword: decision making, uncertainty, information fusion, logics, uncertain information processing, computing with words
Categories: I.1, I.2, M.4, F.4.1

1 Introduction

Decision making is always related to humans' intelligent activities. Uncertainties involved in the objective physical world are associated with the subjective humans' mental activities, such as, randomness [Nillsson 1986], fuzziness [Zadeh 1965], indetermination [Deng 1982; Wang 1990], indistinguishability [Pawlak 1982; Zhang *et al*. 2003], incomparability [Xu *et al*. 2003], incompleteness [McDermott *et al*. 1980], and incredibility [Liu *et al*. 2004] etc. Humans' mental activities are thus involved in uncertain information processing. Decision making based on uncertain information is called decision making under uncertainty.

In real world decision situations human beings often face the following issues: 1) classification of uncertain information; 2) integration of uncertain information; 3) uncertain information process based on certain criteria/rules; 4) uncertainty reasoning and decision judgments; 5) the use of linguistic variables in natural language to carry out uncertainty reasoning because of uncertain description. The paper addresses each one of the previous five issues and raises not only some open but also noteworthy research problems. The paper is organized as follows: in Section 2, a short overview of different types of uncertainties is provided by analyzing the sources of them. Typical theories and methods on uncertain information fusion are discussed from different point of views in Section 3, along with some open reseach issues raised. Section 4 provides a short review about logic based uncertain information processing and a review about logic based decision making under uncertainty is given in Section 5; Section 6 focuses more details on decision under uncertainty based on linguistic information representation. The paper is concluded in Section 7.

2 Classification of Uncertainties

Generally speaking, real world problems involve different types of uncertainty To clarify the types of uncertainty involved a first critical issue is to determine what kind of theories or methodologies can be adopted to solve the problem accordingly and effectively. Hence, a brief overview of some types of uncertainty is provided by analyzing the sources of those uncertainties.

Randomness is a kind of uncertainty indicating if an event will happen or not under certain condition. It is caused from the fact that, the conditions, which effect the occurrence of an event, are too loose in order to be able to determine the causal relationship of the events – a collapse of “law of causation”, which means also that the looseness of conditions will determine the degree of randomness. The stronger for the condition of restriction, the less for randomicity. For example, randomness could be classified as complete certain, basically certain and complete uncertain for three cases: stome frusta, narrow bridge and wide bridge respectively.

Fuzziness is a kind of uncertainty reflecting the difference of degree among attributes of each intermediate variation state of objects. It is caused from the feature of the object belonging to this and that at the same time – a collapse of “law of excluded middle”, which is normally reflected by the “membership degree” of the objectcs belonging to a concept. For example, “old” for the ages “5 years old”, “60 years old”, and “100 years old”.

Indetermination is a kind of uncertainty reflecting on human’s subjective understanding including both the determinate and indeterminate part due to the limited conditions, which are shown normally by the “accuracy degree” of things understood by human beings.

Indistinguishability is a kind of uncertainty reflecting the difference and similarity among different objects. It is rather hard to distinguish each other when the object is observed roughly or measured by a rough scale; and it is much easier when the object is observed delicately or measured by the fine scale. Hence, it is normally reflected by the “precise/fine degree” of the measurement scale to the objects. Considering an example of student exam marks in [0, 100], students’ mark can be complete partitive if it is divided into 101 levels as 0,1,2,…,99,100; basically partitive if it is divided into bad -[0, 60]; good -[61, 85]; excellent [86, 100]; complete non-partitive if it is divided into only 1 level as {0,…,100}.

Incomparability is an important type of uncertainty, especially inevitable in decision making and evaluation situations, but it is not easily handled through conventional methods because of its complexity. It is caused by the complex degree of an object. The more complex for certain object, the more for attribute of certain,, and the larger for incomparability. Taking an example of student exam marks for different subjects {Mathematics, Physics, English, Chinese}, the marks for student 1 could be {90, 70, 85, 95} for each subject respectively, and the marks for student 2 {70, 90, 95, 85}. Only one subject is easy to compared, more subject lead to more incomparability.

When human beings try to understand and deal with practical problems, especially in their decision making process, comparison is a way commonly used to tell something about something else – "there can be no differentiation without comparison". Some claim that chains, i.e., totally ordered sets, can be applied in most cases. But very often the assumption is an oversimplification of reality; in fact, relations in the real world are rarely linear. Incomparability is a kind of uncertainty often associated with humans' intelligent activities in the real world, and it exists not only in the processed object itself, but also in the course of the object being processed. It is a kind of overall uncertainty of objects due to the complexity of objects themselves, associated with many factors and the inconsistency among those factors. This fact implies an overall uncertainty of objects, which can be due to missing information, ambiguity or conflicting evaluations.

Incompleteness is a quite common type of uncertainty when it is difficult or impossible to fully understand some objects because they cover too broad ranges beyond our capability either the resources or conditions are limited. Therefore, this kind of uncertainty is normally reflected by the "completeness degree" of the objects to be understood by human beings.

Incredibility is also a kind of uncertainty caused due to the complexity of the objects. It is normally rather difficult or impossible for human beings to very deeply and truly understand something, instead, in most of cases it is easy to find the deformation or distortion of the information about it, because there are always complexity, different types of stages, and a wide range of subjects in its course of development. Therefore, this kind of uncertainty normally is reflected by the "depth" of the objects to be understood by human beings.

3 Uncertain Information Fusion

The integration of uncertain information has been studied and coped with different points of view depending on the framework where it happens.

3.1 Complexity of human environment

In general, the environment for human life is very complex, which is manifested mainly in the following four aspects:

(1) Complexity according to the type of uncertainty. We often face problems or systems with multiple objects, attributes, levels, and factors which lead to multiple types of uncertainty and even inconsistency;

(2) Various ways of representing uncertainty. Uncertain information can be represented as numerical values, symbols, natural language, tables, graphs, categories, etc.

(3) Different uncertainties not only exist in the processed object itself, but also in the course of the object being processed.

(4) Various dynamic features reflected in uncertainty with the changes of time, space, conditions, etc.

Therefore, it is necessary to rationally integrate all this complex uncertain information (called uncertain information fusion) to make a rational decision.

3.2 Typical theories and methods on uncertain information fusion

Different theories and methods regarding uncertain information fusion, which either aimed at numerical based information or information being able to be transformed into the numerical forms, have been proposed, such as [Yager 1993; Yager and Kacprzyk 1997; Herrera *et al.* 1997; Marichal 1998; Calvo *et al*. 2002; Ruan 2002; Xu and Da 2003; Beliakov *et al*. 2007].

There have been also some researches about other types of uncertain information, but not extensively investigated yet. Since it is often inevitable to deal with uncertain information which is non-numerical, and difficult or impossible to transform it into any numerical form, such as non-numerical information fusion, hence, it is necessary and important to investigate theories and methods on non-numerical information fusion, which are further detailed in the following subsections.

3.3 Open research issues in uncertain information fusion

A variety of theories and methods have been proposed on uncertain information fusion, however, many important issues still remain to be investigated extensively:

(1) How to effectively aggregate natural language based information

In many real situations, natural language is a much better tool to naturally describe uncertain information than precise values. In terms of natural language based uncertain information fusion, most of the existing approaches normally start quantifying uncertain information represented by natural language into numerical forms, then implement numerical information fusion. In general, quantifying natural language information either causes the loss of information or inaccurate fusion results due to the improper quantification approach. Moreover, numerical fusion results are often far away from natural language fusion output. Therefore, it is necessary and important to investigate theories and methods for direct and effective natural language information fusion without necessity of any transformation in order to reduce or avoid any loss or distortion of information

(2) How to effectively aggregate graph-based information

In general, graphs (figures or pictures) represent uncertain information in a richer and more visual way as well as it is easier to understand than numerical values. The existing fusion approaches for this kind of information often follow an scheme that first transforms the graph information into numerical values, then proceeds the numerical fusion. Again it causes loss or distortion of information. Therefore, it is also necessary and important to investigate theories and methods for direct and effective graph-based information fusion in order to reduce or eventually avoid those limitations.

(3) How to effective aggregate symbol-based information

Appropriate symbols can effective and intuitively represent some uncertain information. Similarly to natural language and graph information, it is also necessary and important to investigate theories and methods for direct and effective symbol-based information fusion to avoid any loss of information.

(4) How to more effective and properly aggregate different types of uncertain information

In general, there are intrinsic differences between different types of uncertain information. So it is very difficult and challenging to appropriate and effectively aggregate them into some generic information format. The current proposals showed the way to transform them firstly into a unified information space, and then proceed the aggregation within this unified information space. In most of cases, it is very difficult either to find an appropriate unified information space or find the appropriate and effective transformation approaches. But it will be still worthwhile to explore this direction on the one hand; on the other hand, it seems quite important and promising to investigate direct fusion theories and methods for different types of uncertain information without any previous transformation.

(5) How to more effective and properly aggregate inconsistent uncertain information

In general, the incompleteness in the information system under uncertainty often leads to the inconsistency between different uncertain information within the system. In order to aggregate the inconsistent uncertain information, the common adopted approach is so-called “compromise” method – where the roles of various types of uncertainty are reflected in the aggregated information (e.g., adopted the weighted average approach in some situations), this actually requires the investigation of theories and methods about how to rationally achieve this kind of “compromise”.

4 Logic Based Uncertain Information Processing

Human beings always manage the processing of uncertainty based on different principles according to its type. From the point of view of symbolisms, these principles are formalized into the corresponding logic systems, and it is highly necessary the study and establishment of the logical foundation for uncertainty reasoning analogous to the way in which classical logic provides a foundation for certain reasoning. As pointed out by Zagare ([Brown *et al*. 2000], p.103): “Without a logically consistent theoretical structure to explain them, empirical observations are impossible to evaluate; without a logically consistent theoretical structure to constrain them, original and creative theories are of limited utility; and without a logically consistent argument to support them, even entirely laudable conclusions… lose much of their intellectual force”. It means that only across the exploration of the underlying logic we can ascertain the consistency and completeness of our analyses. Up to now, various kinds of non-classical logic systems have been extensively studied in the context of finding natural and efficient inference systems to deal with uncertainty.

Non-classical logic has become a key formal tool for computer science and artificial intelligence, some of which are beiefly overviewed as follows:

A. Probabilistic logic is normally adopted for stochastic information processing. Nillsson (1986) introduced a formal probabilistic logic based on probability theory and mathematical logic, where the probability of a proposition is linked to the truth-value of the proposition. Probabilistic logic provides a logical foundation for stochastic information based uncertainty reasoning.

B. Fuzzy logic has been adopted for fuzzy information processing. Since Zadeh introduced "Fuzzy Sets" in 1965 [Zadeh 1965], it has been investigated and developed intensively, and applied to different areas, particularly in artificial intelligence. In the aspect of formal logic system, many researchers devoted themselves to the formal deduction, the correlation between semantics and syntax within the related many-valued logic and lattice-valued systems [Goguen 1967; Pavelak 1979; Novak 1982 and 1989; Bolc and P. Borowik 1992; Hajek 2000; Novak *et al*. 2000; Cignoli *et al*., 2000; Gottwald 2001; Xu *et al*. 2003]. These research results have provided important theoretical foundations on formally handling fuzziness in a deeper, more effective and standard way to establish the fuzzy reasoning theory and methods based on a strict logical system.

C. Indetermination theory of mathematics has been investigated to handle indeterminate information. Wang (1990) introduced the concept of indeterminate number and proposed a mathematical method to deal with indeterminate information from the viewpoint of soft design in engineering. Afterwards, indetermination mathematics was introduced by [Wang and Wu 1992] together with some systematic indeterminate rational number theory and its applications [Liu *et al*. 1997; Yue *et al*. 2001].

D. Quotient space theory has been investigated to deal with indistinguishability. Zhang *et al*. (2003) proposed quotient space based on granular computing. In addition, rough logic was also proposed to deal with the indistinguishable information [Nakamura 1996; Duntsch 1997; Zhang 2001], which characterizes approximate distinguishability of the investigated objects and has been developed as a systematic theoretical foundation in decision analysis.

E. As for incomparability, an important algebraic structure – lattice (or lattice structures) has been very useful and provides a well-developed branch of abstract algebra for modelling the ordering relations in the real world, and the general framework of lattice theory is almost indispensable in explaining complex phenomena in an easy way [Birkhoff 1967; Montero 1995]. Lattice-valued logic, as one of the most important many-valued logics, extends the chain-type truth-valued field to a general lattice. Where the truth-values characterized by general lattices are incompletely comparable with each other can provide an alternative logical ground and approach for dealing with imprecision, especially incomparability. Many researchers devoted themselves to the formal deduction, the correlation between semantics and syntax within the related lattice-valued systems. In the 1930s, Jordan *et al*. (1934) and Biskhoff (1936) (futher details in [Svozil 1998]) introduced an important lattice-valued logic – quantum logic based on orthomodular lattices in order to provide an axiomatized foundation for quantum mechanics. By extending the fuzzy set concept into lattice-valued (L-valued) fuzzy sets, Goguen (1967) studied lattice-valued logic and proposed the first lattice-valued logic formal system based on complete lattice-ordered semigroups, which provided a new instrument and approach to study the lattice-valued logic. Thereafter, Pavelka (1979) made some improvements and developments on Goguen's logic system, incorporated internal truth value in the language, established a fuzzy propositional logic system whose truth value set is an enriched residuated lattice and proveded many important results about its axiomatizability. Pavelka's work is mainly concerned with propositional fuzzy logic. Novak (1982) extended it to the first-order fuzzy logic. Pavelka and Novak's work provided a relative general frame for the lattice-valued logic system. After that, many authors still devoted themselves into fuzzy logic in the light of Pavelka and Novak's works, (*i.e.*, many-valued logic based on residuated lattice, such as [Turunen 1995; Esteva *et al*. 2000; Novak 1989; Hajek 2000; Wang 1999 and 2004; Ying 1993 and 2001; Novak *et al*. 1999; Cignoli 2000; Gottwald, 2001; Xu *et al*. 2003], where many of those important and remarkable results reflect fundamental characteristics of fuzzy logic.

Among the existing methods exploiting the structure of the lattice-valued algebra of truth values, we highlight [Morgan *et al*. 1976; Orlowska 1978], which gave a theorem proving systems for *m*-valued Post logics and for algorithmic logics. Belnap (1977) proposed a 4-valued logic that also introduces the value 'both' (*i.e*., "true and false"), to handle inconsistent assertions in database systems. Salzer (1996) studied operators and distribution quantifiers in finite-valued logics based on semi-lattices, and Hahnle (1998) derived tableau-style axiomatizations of distribution quantifiers by using Birkhoff's representation theorem for finite distributive lattices. Another kind of particular lattice-value logics is the logic based on bilattice. It is a natural generalization of classical two-valued logic, which is introduced by Ginsberg in [Ginsberg 1987], and more completely in [Fitting 1988, 1989; Ginsberg 1988]. Sofronie-Stokkermans (1997) discussed the finite-valued logic. Liu (1980) proposed a logic system based on a complete lattice with the dividing element. Since 1997, Wang investigated the logic system based on certain class of algebras and also obtained some important results [Wang 1999 and 2004]. To provide a logical foundation for uncertain information processing theory, especially for the fuzziness, the incomparability in uncertain information in the reasoning, Xu (1993) established lattice implication algebra by combining lattice and implication algebra and the corresponding lattice-valued logic systems. Uncertainty reasoning methods and the resolution-based automated reasoning based on these logic systems were established as well in [Xu *et al*. 2003]. The aforementioned works for many-valued logics and lattice-valued logics have extended classical logics from different points, making them an important foundation for the research of many-valued logics,lattice-valued logics and their applications in decision making.

F. Among the research to handle incomplete information, non-monotonic logic and reasoning is one of the typical approaches. Reiter (1978) proposed "closed world assumption" (i.e., a statement is true when its negation cannot be proven). Doyle (1979) proposed "truth maintenance system". MaCarthy (1980) proposed "circumscription" theory, later on, McDemott & Doyle (1982) proposed "modal nonmonotonic logic". Reiter (1980) also proposed "default logic". Moore (1985) extended "modal nonmonotonic logic" and proposed "autoepistemic logic". All these non-monotonic logic placed foundations for different non-monotonic reasoning approaches. Some more recent works on non-monotonic logic and non-monotonic reasoning have been done, *e.g.*, among others [Poole 1988; Lukaszewicz 1990; Pearl 1990; Geffner 1992; Marek 1993; Schlechta 1997; Lehmann 2001], and some applications of non-monotonic reasoning (*e.g.*, [Dix *et al*. 1997; El-Azhary *et al*. 2002; Morgenstern 1998]). Extensive surveys on non-monotonic reasoning approaches can be found in [Ginsberg 1987; Besnard 1989; Cadoli and Schaerf 1993; Donini *et al*. 1990; Gabbay *et al*. 1994; Lukaszewicz 1990]. The typical non-monotonic reasoning methods are all based on the corresponding non-monotone logic system. Up to now, the non-monotone logic system has been a promising direction and attached considerable attention in AI.

G. Credibilistic logic has been investigated to handle unbelievable information. Focusing on information with fuzziness, Li and Liu extended classical binary logic and proposed Credibilistic Logic [Liu 2004 and 2008; Li and Liu 2009], which provided a theoretical support for handling credibility in fuzzy reasoning.

5 Decision Making Based on Uncertainty Reasoning

As overviewed in Section 3, human intelligent activities and decision making always involve dealing with uncertainty and cannot be done without reasoning. Hence, the reasoning is under uncertainty environment – uncertainty reasoning. The type of uncertainty reasoning utilized depends on the uncertainty environment. Reasoning is essentially based on logic, therefore logic-based decision making under uncertainty is desirable.

As for reasoning and decision making under randomness, on the one hand, there exists probabilistic logic based uncertainty reasoning [Nillsson 1986], but no extensive work has been already done on decision making under randomness by applying probabilistic logic based reasoning. On the other hand, there exist uncertainty reasoning approaches based on probability theory [Duda *et al*. 1976; Shortliffe and Buchanan 1975; Dempster 1968; Shafer 1976], in which some expert systems were developed, such as PROSPECTOR (a consultation system for mineral exploration) [Duda, *et al*. 1978 and 1984], Computer-based medical consultation system [Shortliffe 1976]. Some others decision making directly based on probability and statistics theory, i.e., Stochastic Decision (including decision making under risk, decision making under unknown state probability, Markov Decision) and Statistical Decision (including empirical Bayesian decision and Bayesian decision), e.g., [Berger 1985; Robert 2001].

As for reasoning under fuzzy information, a lot of extensive works have been done based on fuzzy logic inference in a broad sense and formal fuzzy logic inference in a narrow sense as well. There are also extensive works on decision making under fuzziness based on broad fuzzy logic theory and approaches. The decision making approach based on formal fuzzy logic is actually a promising direction because logic-based decision making will have more rational theoretical support.

As for decision making under indeterminate information, on the one hand, a few approaches have been proposed based on indetermination theory of mathematics [Yue *et al*. 2001], especially, a concept of indeterminate rational number theory has been utilized in decision making, system optimization, data processing, interval analysis, electronic engineering, and construction engineering etc. On the other hand, some decision making approaches based on grey system theory [Deng 1986; Wang *et al*.

2001] have been proposed, including decision making under grey situation, grey programming, grey hierarchical decision making, and grey comprehensive evaluation. However, the corresponding logical foundations and their reasoning approaches have not been extensively investigated yet, further investigation on decision making under indeterminate information based on indeterminate logical reasoning theory and approaches is obviously a promising but challenge direction to be proceed.

As for decision making under indistinguishability, on the one hand, quotient space theory has provided certain reasoning scheme for decision making, such as application in dynamic programming and temporal programming [Zhang *et al*. 1992 and 2007]. On the other hand there are decision making approaches based on rough logic inference [Pawlak 1991; Nakamura 1996], only some preliminary works have been done on dealing with indistinguishability, more research is expected within the rough logic framework.

As for decision making under incomparability, some approximate reasoning approaches have been proposed based on lattice-valued logic systems [Hajek 2000; Novak *et al*. 1999; Cignoli 2000; Gottwald, 2001; Xu *et al*. 2003; Guo *et al*. 2003], which reflect the research ideas of dealing with incomparable information based on logical inference and lattice-ordered preference structure. It is foreseen that decision making based on lattice-valued logic and reasoning for handling incomparable information will be an important direction in decision making under uncertainty.

As for decision making approach under incomplete information, most of approaches are based on different types of non-monotonic logical reasoning, such as [Bernferhat *et al*. 2001]. In fact, non-monotonic reasoning reflects the essential principle in decision making under incomplete information. Hence, non-monotonic logical inference is still an important research direction for decision making under incomplete information.

6 Decision under Uncertainty Based on Linguistic Information Representation

There is much qualitative information in the area of evaluation processing and decisionmaking, like subjective judgment of experts etc., which cannot be set out in a precise numerical way. Human beings usually express the world knowledge by using linguistic terms in natural language which is full of imprecision and vagueness. A linguistic term differs from a numerical one in that its values are not numbers, but words or sentences in a natural or artificial language. Hence, people use linguistic variables in natural language as a suitable way to carry out uncertainty reasoning.

Instead of extending precise numerical approaches, machine intelligence should be thus based on uncertainty reasoning with words. In this way, the machine can imitate human beings to make linguistic truth values based uncertainty reasoning.

6.1 Representation of linguistic type information

Different computational approaches in the literature addressed Computing with Words (CWW) process. The books of Computing with Words [Wang 2001; Zadeh and Kacprzyk 1996 and 1999] are good collections of papers on various theories and applications of CWW. And some journal papers on CWW, such as [Kacprzyk and

Zadrozny 2001; Walley and Cooman 1999; Garcıa-Cascales and Lamata 2009; Dvorak and Novak 2004; Zadeh 1996, 2008; Lawry 2001; Ying 2002] are good examples of this topic. Membership functions are generally at the core of many fuzzy-set theories based Computing with Words. Apart from fuzzy-set theories based CWW, there exist some alternative methods developed to model and compute with linguistic information in natural language from the different point of view, such as fuzzy ordinal linguistic approach, hedge algebras, probabilistic model of CWW, linguistic-valued information processing mainly based on a logical algebra structure - lattice implication algebras, linguistic labels etc. [Delgado *et al:* 1993; Torra 1996 and 2001; Yager 1988, 1995, 2004; Ho and Wechler 1990; Ho and Nam 2002; Herrera and Herrera-Viedma 1997; Herrera and Martínez 2000 and 2001; Qiu and Wang 2005; Wang and Qiu 2003; Lawry 2001 and 2004; Meng *et al*. 2006; Xu and Da 2003; Liu *et al*. 2005; Li *et al*. 2008; Zou *et al*. 2008; Pei *et al*. 2007], more details can be referred to [Pei *et al*. 2009] and reference therein.

One important issue in representing and dealing with linguistic information is that it can sometimes be difficult to distinguish the boundary of these words in different natural languages, but their meaning in common usage can be understood. Moreover, there are some “vague overlap districts” among some words which cannot be strictly linearly ordered, e.g., notice that *approximately true*, *possibly true*, and *more or less true* are incomparable. This structure cannot be collapsed into a linearly ordered structure, because it would impose an ordering among them which was originally not present. This means the set of linguistic values may not be strictly linearly ordered. Linguistic terms can be ordered by their meanings in natural language. Naturally, it should be suitable to represent the linguistic values by a partially ordered set or lattice. According to this feature of linguistic variables, we need to find some suitable algebra to characterize the values of linguistic variables. There is why this linguistic-valued algebra approach can provide us simple algorithms for reasoning and decision-making. To achieve this goal one possible way could be modeling the set of linguistic truth-values by using the lattice to construct the algebraic structure of the linguistic domain. It is certainly a promising but challenge research direction.

6.2 Algebraic structure to model incomparable linguistic information

Although there have been some investigations on the algebraic structure of linguistic truth values together with applications in decision making and social science, it still lacks a formalism for the development of logic systems based on linguistic truth values, approximate reasoning and automated reasoning based on linguistic truth-valued logic systems as well. Among others, one key problem that has not been studied sufficiently is how to choose a comparatively appropriate linguistic truth-valued algebraic structure, which can provide a comparatively appropriate interpretation for the logical formulae in linguistic truth value logic systems. And accordingly provide a strict theoretical foundation, as well as a convenient, practical, and effective underlying semantic structure to automated uncertain reasoning based on linguistic truth-valued logic, and various kinds of corresponding intelligent information processing systems [Pei *et al*. 2009].

This algebra is based on natural semantic properties of linguistic hedges, the set of which is equipped with a partially ordered relation (we consider lattice ordered because of its computational simplicity and much more rich properties following common sense axioms). This would formally establish a semantics for linguistic truth-valued logic. It is worth noting that, as expected, this kind of algebraic structure should satisfy the following assumptions: (i) it should have a well-defined logical algebraic structure; (ii) linguistic truth values adopted should be consistent with the meaning of commonly used natural language; (iii) linguistic truth values adopted have allowed enough discrimination; and (iv) the set of linguistic truth values should be a modestly small set of linguistic truth values which can cover commonly used natural linguistic expressions. This algebraic structure should not only model the incomparability in the linguistic truth-values, but also characterize the logical relationship between linguistic truth-values, especially the logical implication relationship, in order to stablish a foundation to lattice type linguistic valued logic system for uncertainty reasoning and automated reasoning. There is still extensive work remained to be investigated on the structure, characterization of linguistic truth-valued as well as its rational reflection in representing linguistic information, some work relevent to this direction can found in [Ho and Wechler 1990; Yager 1995; Ho and Nam 2002; Liu *et al*. 2005; Li *et al*. 2008; Meng *et al*. 2006; Zou *et al*. 2008; Pei *et al*. 2009].

The long-term goal of this research is to develop a systematic lattice-based linguistic truth-valued logic system, its approximate and automated reasoning theory, methods and prototype systems. This research direction focuses on a flexible and realistic approach, i.e., the use of linguistic truth-values in natural language, in which the symbolic approach acts by direct computation and reasoning on linguistic terms. The reasoning methods should have, not only a rational logical semantic interpretation, but also strict logical syntactical proof, being able to deal with "two levels" (the object itself and the process) and "two types" (fuzziness and incomparability) of uncertainties [Xu *et al*. 2003].

6.3 Logic system for incomparable linguistic-valued information – lattice-type linguistic truth-valued logic system

There exist a few linguistic truth-valued logic systems, however, some crucial but fundamental research issues still need further and extensive attention, e.g., it would be desirable to investigate the corresponding linguistic truth-valued logic system as well as their specific logical structures and properties according to different contexts or requirements in processing linguistic information, this will place some specific logical foundation for handling different type of linguistic information, some work have been summaried in [Pei *et al*. 2009]. The questions about complex linguistic truth-value structure are still open, we believe that it is feasible and reasonable to use lattice-valued algebra and lattice-valued logic to establish strict linguistic truth-valued logic and various kinds of corresponding linguistic information processing systems, based on what have been done so far about lattice-valued algebra, lattice-valued logic by different researches, also relying on a continuous work on this direction.

6.4 Uncertainty reasoning based on lattice-type linguistic truth-valued logic

Focused on linguistic truth-values, the research in this framework aims at establishing linguistic truth-valued approximate reasoning theories and algorithms so that the output from the reasoning has not only rational logical semantic interpretations, but also strict logical syntactical proofs, which is consistent from the logical point of view. Two possible critical technical paths are: (1) choose lattice implication algebra (LIA) [Xu *et al*. 2003] as an algebraic model of linguistic truth-value domains, that is, an axiomatic system of an algebraic structure that defines and determines how the linguistic modifiers (hedges) are combined with the primary linguistic terms. Therefore, based on the established uncertainty reasoning lattice-valued theories and algorithms with truth-valued in an LIA, the corresponding LIA-based linguistic truth-valued logic and their reasoning theories and algorithms can be established. However, there are still some open issues remained unsolved: the determination of the set of interpretation (including the selection of suitable truth-valued field); the selection of rule sets; the representation of inference output; (2), directly establish linguistic truth-valued logic systems, then investigate their reasoning theories and algorithms, this routine seems even harder than the former one.

6.5 Decision making under uncertainty based on lattice-type linguistic truth-valued logic based approximate reasoning

We hold the opinion that decision making is often associated with reasoning intentionally or not, and reasoning is necessary to have a rational logic basis, so decision can be made based on the rational logical reasoning. In addition, human activities always involved uncertainty. we often carry out uncertainty reasoning within an uncertainty environment and then make judgments and decisions based on this kind of reasoning. Uncertainty reasoning should be based on the rational logic with the capability to handle uncertainty, decision under uncertainty can be made based on the corresponding rational logical reasoning. Furthermore, we often use linguistic variables in natural language as the way to describe uncertainty and carry out uncertainty reasoning. In terms of machine intelligence, the key point is to make the machine imitates human beings' uncertainty reasoning scheme by means of linguistic variables in natural language. The development of science and technology has attempted to investigate humans' subjective logic category based on previous investigations of the objective physical world. Hence, instead of extending precise numerical approaches, machine intelligence should be based on uncertainty reasoning with words so that the machine can imitate human beings to make linguistic truth value based uncertainty reasoning.

7 Conclusions

The use of fusion approaches and logic based processing to deal with uncertainty in decision making under uncertainty have provided successful results in the past. But it is still worthy to investigate new fusion approaches and the logic based reasoning in order to set up a consistent and complete strict theoretical foundation, as well as a convenient, practical, and effective underlying semantic structure to automated uncertain reasoning for decision making. In addition, it is necessary and important to investigate theories and methods on non-numerical information fusion which may be difficult or impossible to be transformed into numerical forms. Therefore it is clear as it has been showed in this paper that there exist promising and challeging research directions on these topics because many problems addressed in this paper have not been satisfactorily solved yet.

Acknowledgements

This work is partially supported by the National Natural Science Foundation of China (Grant Nos. 60474022 and 60875034), the Specialized Research Foundation for the Doctoral Program of Higher Education of China (Grant No. 20060613007), the China-Flanders Bilateral Scientific Cooperation joint project (Grant No. 011S1105), as well as the Research Project P08-TIC-3548 and TIN2009-08286.

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